Examining Watershed Processes Using Spectral Analysis Methods Including the Scaled-Windowed Fourier Transform

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Important characteristics of watershed processes can be extracted from hydrologic data using spectral methods. We extract quantitative information from precipitation, stream discharge, and groundwater head data from watersheds in northern-lower Michigan using Fourier Transform (FT) methods. By comparing the spectra of these data using similar units, we graphically illustrate the hydrologic processes that link precipitation to stream discharge and groundwater levels including evapotranspiration. We also demonstrate how unit hydrographs can be efficiently and non-parametrically derived using the FT in a manner that allows for a quantitative seasonal comparison of precipitation and the resulting stream discharge response. This analysis clearly illustrates the reduction in summer discharge levels due to canopy interception and evapotranspiration. We also develop a systematic application of the FT we call the Scaled-Windowed Fourier Transform (SWFT), which extracts time-varying spectral content using a similar approach to the wavelet transform. While computationally less efficient than the wavelet transform, the SWFT allows for embedded detrending and tapering. Application of this method clearly illustrates the non-stationarities of spectral content within the three chosen data types, leading to a greater understanding of discharge-generating processes.

INTRODUCTION

Spectral analysis (SA) provides a powerful means of extracting information from hydrologic data. This type of analysis can reveal processes that may be obscured in direct time-series analysis by providing data not just on temporal fluctuations, but also on the spectrum of frequencies within those fluctuations. Furthermore, integrating multiple data types in a quantitative and meaningful way is relatively simple using spectral analysis. Here we apply both existing and novel SA techniques to hydrologic data from two watersheds in northern lower-Michigan. We use these methods to illuminate the processes that link precipitation to stream discharge and groundwater levels.

Spectral analysis has been applied within the hydrologic sciences for decades [e.g., Bras and Rodriguez-Iturbe, 1985; Hameed, 1984], most commonly as a means of estimating coefficients for linear autoregressive or stochastic models [e.g., Naff and Gutjahr, 1983; Jukic and Denic-Jukic, 2004; Zhang and Schilling, 2004]. During the last decade, SA has been applied to examine fractal and multi-fractal behavior of systems [e.g., Tessier et al., 1996; Pelletier and Turcotte, 1997; Kirchner et al., 2000], and to examine linkages between hydrologic and climatic processes [e.g., Tessier et al., 1996; Pelletier and Turcotte, 1997; Coulibaly and Burn, 2004].

Here we apply SA techniques to explore linkages between hydrologic processes and to provide a deeper understanding of those processes. Previous SA process comparison studies have generally not assured the similarity of measurement units, nor have they (with the exception of more recent wavelet studies including Gaucherel [2002] and Coulibaly and...
Burn (2004)) considered the time-variant nature of process spectra. We demonstrate the quantitative utility of comparing data and spectra with similar physical units for inference of process relationships. Also, using an application of the common Fourier Transform we call the Scaled-Windowed Fourier Transform (SWFT), we illustrate the non-stationary behavior of precipitation, stream discharge, and groundwater head spectra. Comparing Fourier spectra to the SWFT output, we illustrate how ignoring such non-stationarity can result in misinterpretations of spectra and thus of inferred process details. Finally, using a spectral derivation of seasonal unit hydrographs, we demonstrate how simple stream discharge models can be improved by considering the seasonality of watershed processes.

Recognizing that spectral analysis is not a standard tool in the hydrologic science toolbox, Fleming et al. (2002) published a practical introduction to SA that focused on applications of the Fourier Transform (FT) including direct frequency domain investigation, spectral filtering, and spectral simulation model validation. Here we apply SA in a similar manner, explaining our assumptions and the common complications, and demonstrating how these methods can be used across the hydrological sciences. Additionally, parts of the methods section are intended to contribute to a set of “best practices” of SA in the hydrologic sciences.

**DATASETS AND STUDY AREA**

Six data types from two northern lower-Michigan watersheds were used in this study: daily temperature, precipitation, and snowfall, both 15-minute and hourly stream discharge, and bi-hourly groundwater heads. Figure 1 shows the locations of both Michigan watersheds, as well as the locations of our groundwater transducers. The 3711 km² Evart sub-basin of the Muskegon River Watershed (MRW) was selected for this study because there are no actively controlled flow structures on the Muskegon River or its tributaries upstream of this station.

Because the Evart sub-basin lacks a network of monitored groundwater wells, we used water level data from our pressure transducers in the nearly adjacent Grand Traverse Bay Watershed (GTBW), which has a similar hydrogeological setting. Unconsolidated sediments within the Evart sub-basin and the G TBW were deposited by the same set of glacial episodes. These sediments are characterized primarily by coarse to fine sands and gravels with a small percentage of fine-grained material [Farrand and Bell, 1982]. The similarity in depositional history and topographic variation between these two basins suggests that G TBW groundwater head fluctuations can be used as a proxy for the nearby upper MRW.

![Figure 1](image.jpg)

**Figure 1.** Map of the Muskegon River and Grand Traverse Bay watersheds, with an inset map of Michigan showing their locations. Groundwater wells with transducers are marked with triangles, and the Evart gauge sub-basin is shaded in grey.
hourly interval. NEXRAD data show a high degree of correlation to the corresponding gauge values in this region [Jayawickreme and Hyndman, 2007]. These radar-based precipitation data are only available for approximately 9 months out of the year in this region (from 4/1/2004 to 11/30/2004) due to errors in snowfall estimates, so they are only suitable for evaluating shorter-period system behavior.

We installed a network of 17 pressure transducers in USGS groundwater wells across the GTBW. Water table elevations were recorded every two hours beginning either in June 2003 (9 transducers) or June 2004 (8 transducers). Data through 7/1/2005 were used in this analysis.

METHODS

Fourier Transform

Fourier’s Theorem states that any complex periodic function can be decomposed into a set of periodic basis functions of varying amplitude, period, and phase shift. The most common such decomposition technique is the Fourier Transform (FT), which uses sinusoidal basis functions. Fleming et al. [2002] present the basic theory, and a full mathematical treatment of this technique can be found in textbooks [e.g., Bras and Rodriguez-Iturbe, 1985; Percival and Walden, 1993]. A common implementation of the discrete Fourier Transform (FFT) is the discrete Fast Fourier Transform (FFT) popularized by Cooley and Tukey [1965]. In this study, we use the FFTW libraries that are integrated into the MATLAB computing environment [Frigo and Johnson, 1998]. This particular set of general-radix algorithms does not constrain the user to the requirement in Cooley and Tukey [1965] that the number of samples (N) be a power of 2.

The output of the FFT algorithm is a complex array representing the magnitude and phase shifts of the Fourier coefficients. For instance, the FFT of a sinusoid of unit period and amplitude is an array with a single non-zero value corresponding to a period of 1. The FFT of summation of sinusoids would produce non-zero peaks in the spectrum. If instead the time-series input to the FFT were a broad, single-peaked curve such as a gaussian, exponential, or gamma function, spectral amplitudes would be non-zero across a broad range of periods.

Fourier spectra are generally plotted as power vs. frequency, however we find that plots of amplitude vs. period provide a more natural basis for viewing spectra of interrelated physical phenomena. The spectral amplitude, the square-root of spectral power, of a time-domain input corresponds directly to hydrologic flux quantities such as precipitation and stream discharge. Note that the log-log slope of the amplitude spectrum is $\frac{1}{2} \beta$, where $\beta$ is the slope of spectral power in log-log coordinates. A system is typically considered fractal (or multi-fractal) if its power spectrum roughly follows $1/f^\beta$ behavior [Avnir et al., 1998]. This behavior is typical of a wide variety of geophysical systems, and can provide insight into the processes that govern those systems.

If one assures the similarity of the units of each time-series dataset, the amplitudes of multiple spectra can be compared in a physically meaningful manner. To accomplish this, a suitable unit for comparison must first be chosen. For this study, we chose length/time (L/T) units because precipitation is the most common hydrologic forcing mechanism. In order to change the volumetric discharge units (L$^3$/T) of stream discharge to L/T units, the discharge was divided by the drainage area of the watershed upstream of the gauge. Groundwater head data, measured in L units can be differentiated to yield L/T units. Using the differentiated data, an approximation of the rate of head relaxation at the well location can be obtained by selecting only the negative values. Note, this technique assumes constant lateral inflow. This was then multiplied by an estimate of drainable porosity for the sediments (0.2), to correct for water level differences in porous media vs. open water.

ASSURING PERIODICITY

The FFT algorithm assumes that the data are periodic, namely the starting and ending points of the dataset are identical. Violating this condition results in spurious features in the output spectrum [Bach and Meigen, 1999]. However, time-series data from environmental systems rarely satisfy this criterion. There are four primary means of assuring periodicity: taper function multiplication (tapering); trend subtraction; data subset selection (discussed in “Reducing Aliasing and Leakage” below); and filtering which is not considered here, but the interested reader is referred to Fleming et al. [2002] for a discussion.

Tapering can be a valid means of forcing periodicity if one considers how it affects the resultant spectra. Tapering refers to the multiplication of a mean-removed signal by a “tapering function” (sometimes referred to as a windowing function) that smoothly tapers from a peak of 1 at its center to 0 at the edges. There are a variety of pre-defined tapering functions [Blackman and Tukey, 1958; Harris, 1978], and each affects the spectra differently. Tapering has the side effect of reducing the amount of information in the signal, thus limiting its applicability for short data records [Fleming et al., 2002]. The shape of the tapering function, and how gradually it tapers near the edges, controls how much information is lost in the process. There is a tradeoff since the tapering functions that reduce information loss have more severe spectral “side lobes”, which distort
the transformed spectra by shifting power from primary frequencies to harmonics (integer multiples or factors) of those frequencies. To recover the amplitude of an isolated peak within a spectrum (but not the entire spectrum itself), spectra from each tapering function can be corrected to account for the effect of side lobes. The multiplicative correction factor for each tapering function is given by the ratio of unaltered- to tapered-amplitude [Harris, 1978]. In this study, tapering is primarily applied within the SWFT method where distortion is minimal since only a single amplitude is selected from each FT.

Trend removal is often necessary for environmental data series to assure periodicity while minimizing loss or side-lobe distortion from tapering. The simplest method is to linearly remove the trend from the data, which can be effective if the non-periodicity of the data is associated with a nearly linear trend. If there is some roughly sinusoidal long period fluctuation, a more valid means of trend removal may be to subtract a half-period sinusoid from the signal. In this case, the peak and trough of the subtracted function occur at each end of the original signal. A variety of additional trend-removal techniques have also been developed [Mann, 2004]. We used the sinusoidal trend removal method in this study due to its simplicity and physical basis.

REDUCING ALIASING AND LEAKAGE

If the FT is blindly applied to data without thought to dominant system processes, sampling rates, or sampling interval, aliasing and leakage may occur. Both aliasing and leakage act to shift spectral power (or amplitude) from “true” frequencies to harmonics of those frequencies, although each acts differently. Aliasing results from under sampling high-frequency fluctuations [Bras and Rodriguez-Iturbe, 1985], while leakage or overspill is caused by both the non-periodicity of the system as well as non-periodicity of the processes within that system [Bach and Meigen, 1999]. It is important to note that leakage will occur if the endpoints do not match, but the inverse is not always true. Even if the time-series endpoints match, leakage may occur due to variability of the processes that contribute to the sampled time-series.

In theory, aliasing can be avoided by merely increasing the sampling rate until the time series is fully resolved. Specifically, one cannot resolve spectral peaks with frequencies greater than half the sampling rate (the Nyquist frequency) [Bras and Rodriguez-Iturbe, 1985]. If a time-series has significant power in frequencies above half the sampling rate, aliased power will be present in the empirical spectrum. In the case of many environmental datasets, aliased peaks may not be significant because, as is shown below, these datasets are typically strongly damped at high frequencies. Our analysis indicates that some data do require sampling frequencies on the order of once per hour, but most of those discussed here are sufficiently sampled with daily sampling rates.

Spectral leakage can be more persistent and troubling than aliasing because periodic processes within a system are rarely sampled over an integer number of cycles. Leakage is commonly reduced by applying a tapering function to the time-series, detrending the time-series, or both [Fleming et al., 2002]. However, when the entire FT spectrum is of interest rather than specific spectral peaks, a more effective means of decreasing leakage in environmental datasets may be to carefully select subsets of the data that correspond to natural breaks in processes, thus assuring near-integer sampling without distortion.

Data subset selection is also important because most environmental processes are non-stationary, thus each occurrence of a process may vary in both period and amplitude. If one includes multiple cycles of a periodic process in the FT, the true peak location, shape, and amplitude can be obscured by differences in system states across cycles. Additionally, selecting a subset of data in which some system processes are inactive can also greatly simplify spectral analyses and reveal the spectra of weaker processes in portions of datasets that would otherwise be obscured by dominant, but intermittent, processes. For example, the diurnal fluctuation of stream discharge is often clearly visible during baseflow conditions but can be obscured by runoff and near-stream groundwater discharge during late spring and early summer. Alternatively, if one were only interested in the spectral behavior of runoff, for instance, then selecting data surrounding an isolated moderate-precipitation event during a dry season yields a stream discharge response primarily to direct precipitation and runoff.

UNIT RESPONSE FUNCTIONS

While there are various techniques to derive unit hydrographs from discharge and precipitation data, most are either ill-posed or require assumptions about system behavior [Yang and Han, 2006]. But direct FT deconvolution can produce unit hydrographs quickly and deterministically. The total time-series response of a linear system to a forcing input can be derived from the convolution of the system unit response and the input time-series as follows [Smith, 1997]:

$$ q(t) = \int_{0}^{\infty} h(\tau)p(t-\tau)d\tau $$

where $q$ is total time-series response (i.e. stream discharge), $h$ is the unit response (for the case of stream discharge, this unit response has the special name of “unit hydrograph”),
and $p$ is the input precipitation time series. According to the convolution theorem,

$$Q(k) = H(k)P(k)$$  \hspace{1cm} (2)$$

where $Q(k)$, $H(k)$, and $P(k)$ are the FTs of $q(t)$, $h(t)$, and $p(t)$, respectively, where $k$ is the spectral frequency. The unit hydrograph time-series is then

$$h(t) = F^{-1}\left[\frac{Q(k)}{P(k)}\right](t)$$  \hspace{1cm} (3)$$

where $F^{-1}[\cdot](t)$ denotes the inverse Fourier Transform. Thus the unit-response hydrograph is given by the inverse FT of the ratio of the discharge and precipitation spectra.

Though not strictly necessary, the analysis is simplified if the sampling frequency and units of the precipitation and discharge time-series are identical. The data should be resampled so that the number of samples, $n$, and the sampling frequency, $f$, are equivalent. The unit-response function, $h(t)$, has length $n$, however, in this case the unit response function is only valid up to the point where it becomes negative, since precipitation can not directly produce a decrease in stream discharge \cite{Yang and Han, 2006}. The negative response is therefore the signature of some other watershed process. If baseflow separation is used, this issue can be avoided, though some small uncertainties will remain due to the data themselves, and may result in negative calculated unit responses. Here we chose to not use baseflow separation, because this technique produces a synthetic dataset that is not directly tied to watershed processes and may introduce artifacts of the separation technique that could mask the watershed processes under investigation.

The resultant unit-response hydrograph is not an invariant property of the watershed, as it is sensitive to variations in runoff-generating processes. These processes can be studied by directly comparing different unit-response hydrographs. Differences in response curve timing, peak, and shape can all be used to infer the activity and relative influence of various watershed processes. Applying data subset selection with these process differences in mind can allow for a quantitative sensitivity analysis of system sub-processes.

In this study, we compare seasonally derived unit-response hydrographs for the Evart sub-basin averaged over 10 consecutive years. The derived unit hydrograph will be incorrect if stream discharge is still responding to precipitation inputs that occurred prior to the start of the data period \cite{Smith, 1997}. However, applied over entire seasons this error, as well as any error resulting from noisy data, is greatly reduced. Nevertheless, the derived unit-response function remains highly sensitive to edge conditions of the time-series inputs, thus the nominal time period (given by Table 1) of each season was adjusted to remove precipitation events or sudden increases in discharge near the edges of the time-series. The nominal time period for each season does not correspond to starting and ending dates of each season because they were chosen to assure similarity of hydrologic response within a season based on assumptions of process activity, also listed in Table 1. Also note that the fall and winter seasons overlap because the minimum length of the time-series subset must be longer than the watershed response time, which in this case was on the order of 60 days.

To assure periodicity for the FT of each season’s data, a Tukey tapering function \cite{Blackman and Tukey, 1958} was applied with relatively steep taper (coefficient of 0.1). Tapering was chosen over trend-removal because the magnitude of the discharge response to precipitation was of primary importance. After making these adjustments, some seasons continued to produce non-physical results (characterized primarily by sinusoidal unit-response behavior) and were thus omitted. These omissions are justified on the grounds that the non-physical results reveal only the sensitivities and limitations of the method, and nothing about watershed process.

**SCALED-WINDOWED FOURIER TRANSFORM**

A key difficulty in applying the FT to environmental datasets is that non-stationarities in the data introduce artifacts in the Fourier spectrum. Here there are two types of

<table>
<thead>
<tr>
<th>Season</th>
<th>Nominal Time Periods</th>
<th>Time Period Justification</th>
<th>Years Omitted</th>
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1 Only four years were included in the spring average discharge response, 1991, 1992, 2000, 2001.
non-stationarity to consider. The first is non-stationarity of process period, where subsequent cycles of a process within the system have slightly different spectra due to changes in system properties. The second is best described as intermittency, where a process may be active during only a portion of the sampled data. The first results in spreading of spectral power about a central period (if the process is sinusoidal), while the second results in spurious spectral power at harmonics of the primary period.

To avoid these effects and more clearly illuminate important changes in the spectra, we developed a method that we call the Scaled-Windowed Fourier Transform (SWFT). The SWFT is very similar to a sinusoidal wavelet transform (WT), but it differs in a number of respects. The mathematical development of the SWFT presented here is fundamentally different from that commonly presented for the WT, in a way that may increase the utility of this method for hydrologic scientists. In particular we feel that there is great value in demonstrating that the SWFT produces time- and frequency-localized Fourier coefficients, as opposed to the similarly localized WT coefficients that are wavelet-dependent. Additionally, the SWFT is capable of embedded detrending rather than relying on tapering alone to reduce leakage, potentially producing better spectral estimates. Finally as developed, the periods and times queried by the SWFT are more flexible than typical WT schemes, with the tradeoff of decreased computational efficiency.

The SWFT also differs from the traditional windowed FT that only transforms data within a specific subset of the overall time series called a data window (here the window is different from a tapering function). This data window is then slid along the time series to produce a map of spectral power varying in both period and time. Unfortunately, the windowed FT forces a choice between severe aliasing of low-frequency components of the signal or poor resolution of high-frequency non-stationary processes [Torrence and Compo, 1997].

By contrast to the windowed FT, the SWFT method scales the width of the window over successive passes along the time-series. At each window position, the data are detrended, multiplied by a tapering function, and Fourier transformed. A single amplitude corresponding to the Fourier coefficient of a single frequency is selected from the complete FT at each window position. The window is then slid along the data, producing a time-varying series of amplitudes for that frequency. When the end of the dataset is reached the window width is rescaled and the process is repeated for the next frequency. Note, hereafter we use the word “period” solely to indicate the spectral period, or the inverse of frequency. We feel that the period of spectral content is generally more applicable to the hydrologic sciences than the frequency.

The SWFT produces the same type of scalogram as would be generated with the WT. These scalograms are well defined mathematically and physically (if proper units are used), and can be examined using standard statistical techniques. Aside from simple examination of the scalogram, comparisons of related scalograms such as the cross-scalogram and the coherence phase map (see Torrence and Compo [1997]) can be calculated as well.

Conceptually, the SWFT scalogram is produced via the following:

$$F_{pq} = C \cdot F\left(\{x(j_{\text{start}} : j_{\text{end}}) - D\} T\right)_k$$  \hspace{1cm} (4)$$

where $p$ and $q$ are indices defined mathematically below, which correspond to the periods and times at which the SWFT is applied; $F(\ldots)_j$ indicates a single value from the discrete Fourier Transform spectrum; $x$ is the time-series dataset; $j_{\text{start}}$ and $j_{\text{end}}$ are the beginning and ending indices of the current data window; $T$ is a tapering function (optional); $D$ is a detrending function (optional), each defined over the current data window; and $C$ is a multiplicative correction factor unique to each type of tapering function that is computed as $C = \left|F\left(\sin(-\omega \cdot \pm\infty)\right)\right| / \left|F\left(\sin(-\omega \cdot T)\right)\right|$, where $k$ is the index that corresponds to the period $2\pi$. Note that a reasonable approximation of $C$ can be obtained using just three or four cycles of the sine function. If a tapering function is not used, $C = T = 1$. For our analysis, we chose a Tukey window with a gradual taper (Tukey window coefficient of 0.5) as a tradeoff between frequency resolution and side lobe distortion, resulting in $C = 1.33$. Tukey coefficients closer to 1 produce greater side-lobe distortion, while those nearer to 0 reduce side-lobe distortion but increase leakage.

The complex definition of the discrete Fourier Transform (DFT) modified from Press et al. [1992] is

$$F_k = \sum_{j=1}^{n} x_j e^{-2\pi i (k-1)/(n)}$$  \hspace{1cm} (5)$$

where $x$ is the time-series dataset, and $k$ and $j$ are indices running from $k=[1,...,n]$ and $j=[1,...,n]$ and $n$ is the total number of data points to be transformed. The DFT spectrum $F$ has $n$ points of which the first is the “gain” term, and the next $n/2$ points correspond to the periods $n/f \cdot [1,1/2,1/3,...,2/n]$.

To compute the SWFT, we would like to extract a single Fourier coefficient corresponding to a certain period, $P'_p$ from the entire DFT spectrum. Also, we need the coefficient not for the entire dataset $x$, but for a windowed subset $x$, with indices $j=[n_{\text{start}} : n_{\text{end}}]$ (which relate to times $t = j/f$, where $f$ is the sampling frequency of the dataset) where the total number of points in the window is $N'_p$. To minimize leakage, the window
width $N_p$ should be chosen such that an integer number of sinusoidal cycles fit within it, which can be expressed as

$$N_p = P_p \cdot f \cdot w. \tag{6}$$

Here $w$ is an integer multiple with possible values $[1,2,\ldots,\text{floor}(n/P_p \cdot f)]$ where “floor” is a function that rounds the argument toward zero. In general, higher values of $w$ increase the frequency resolution of the SWFT while decreasing the temporal resolution. Note that the entire dataset is slid along the dataset producing when $w$ is set equal to $n/P_p \cdot f$.

When Equation 5 is applied to the data window $N_p$, we note that the indices $k$ in the DFT output then correspond to periods $N_p/f : [1,1/2,1/3,\ldots,2/N_p]$, and the first point in the DFT output is the DC gain term. The SWFT requires only the value $F_k$ corresponding to $P_p$. So $P_p = N_p/f \cdot 1/w$, and therefore (replacing $k$ with $p$) we get

$$F_p = F_{p-1}. \tag{7}$$

Substituting Equations 6 and 7 into 5, and recalling the complete definition of the SWFT:

$$F_{pq} = C \sum_{j=n_{start}}^{n_{end}} x_j e^{-i \frac{2\pi}{N_p} (j-1)} e^{-i \frac{2\pi}{N_p} (m-1)} \tag{9}$$

where $m$ is an index $[1,2,\ldots,P_p \cdot f \cdot w]$, or $m = (j+1) - n_{start}$, and $n_{end}$ are both functions of $p$ and $q$:

$$n_{start} = (q-1) \cdot P_p \cdot f \cdot w + 1 = (q-1) \cdot N_p + 1, \quad \quad \tag{10}$$

$$n_{end} = n_{start} + N_p - 1. \tag{11}$$

Because the Fourier coefficients can only be time-localized to a window of width $N_p$, we include the capability for each data window to overlap the previous one in order to increase the temporal resolution of the transform. As an example, if $N_p=10$ and no overlap is allowed, the minimum temporal resolution at this value of $P_p$ would be 10. Instead overlap is allowed such that the minimum resolution can be as low as 1. This could either be done with a fixed overlap (i.e., each window overlaps half of the other across all periods), or the overlap can be scaled in any other fashion, such as linearly with $P$. For this study, we define the overlap value, $o_p$, to be given by a simple linear scaling as

$$o_p = \text{floor}\left[\frac{P_p}{\text{max}(P)}(q_{\max} - q_{\min})\right] + q_{\min} \tag{12}$$

where $q_{\max}$ and $q_{\min}$ are specified by the user, and max$(P)$ is the maximum value within the array $P$. The minimum value of $o_{\min} = 1$, and the maximum value of $o_{\max} = N_p$. With this modification, Equation 10 becomes

$$n_{\text{start}} = (q-1) \cdot N_p/o_p + 1 \tag{13}$$

and $q(\max)_p$ is given by

$$q(\max)_p = \text{floor}\left[\frac{n-N_p}{o_p} + 1\right] \tag{14}$$

The amplitude or power spectra can be extracted from the full SWFT spectrum in the same way it would be for the standard FT. Here we use the amplitude spectrum that is calculated via

$$A_{pq} = 2 \left| F_{pq}\right|/N_p \tag{15}$$

where the vertical bars indicate the magnitude of the complex value $F_{pq}$ and the factor of 2 arises because the DFT spectrum is symmetric about the vertical axis and thus distributes half of the spectral power at a period to the each of the positive and negative instances of that period. The center-window times $\tau$ associated with the arrays $F$ and $A$ are given by

$$\tau_{pq} = f \cdot \frac{\left(q-1\right) \cdot N_p}{o_p + \frac{N_p}{2}} = f \cdot \left[ N_p \left(\frac{q-1}{o_p} + \frac{1}{2}\right)\right] \tag{16}$$
Note that $F$, $A$, and $\tau$ are, in general non-rectangular arrays (the exception is when $w=N$). MATLAB’s cell array capability was used to store these values. For convenience of both visualization and further processing, such as contouring or computing cross-scalograms and phase-coherence maps, these arrays can be interpolated to rectangular grids.

The SWFT array $F$ can either be computed directly via equation 9, or it can be computed via equation 4 using the values of $I_{\text{start}}$, and $I_{\text{end}}$ from equations 13 and 11. In either case, the tapering and detrending functions $T$ and $D$ are calculated using the data $x(I_{\text{start}}:I_{\text{end}})$.

The SWFT was developed primarily for flexibly visualizing the non-stationary spectral content of a time-domain signal. Including the ability for $o_p$ to scale with period greatly reduces the computational demand by demanding $F_{pq}$ less frequently for longer periods. Allowing $w$ to vary enables flexibility between time- and frequency-localization, as the needs of the user demand. Though not used here, $w$ could also be a function of $p$ allowing the frequency resolution to also scale with the period. Including explicit tapering and detrending further improves the ability of the SWFT to represent dynamic spectral content when $P$ spans several orders of magnitude.

In order to illustrate the interpretation of the SWFT scalogram, we examine a simple test case using a summation of three separate sinusoids given by: $f(x) = f_1(x) + f_2(x) + f_3(x)$, where $f_1(x) = \sin(4x)\over 0 \leq x \leq \pi$, and $f_2(x) = \sin(10x)\over 0 \leq x \leq \pi$, and

$$f_3(x) = \begin{cases} 0 & 0 \leq x \leq 4\pi, 8\pi \leq x \leq 12\pi \\ \sin(x) & 4\pi < x < 8\pi \end{cases}$$

Figure 2a illustrates the successive superposition of the three sinusoids, the darkest curve plots $f_1(x)$, the mid-tone curve plots $f_1(x) + f_2(x)$, and the lightest curve plots $f(x)$.

The SWFT scalogram (Figure 2b) of the function $f(x)$ reveals the periods, amplitudes, and ranges of activity of each of the simple sinusoids. The method extracts the periods of the three sinusoids and reconstructs the amplitudes accurately, except for the $f_1(x)$. This is simply because only two cycles of the sinusoid were used in $f_1(x)$, and the window width was chosen as twice the period, thus only the very center point should reach an amplitude of 1. The shorter-period sinusoids both have peak amplitudes very near 1, although the middle has amplitudes <1 in some locations because of the interaction between the two longer-period sinusoids. Importantly the longest period sinusoid, which is defined only for $4\pi < x < 8\pi$, only has large amplitudes in this range, thus revealing the time-varying spectral behavior of the input signal.

In cases where the input is something other than a summation of sinusoids, the scalogram output will exhibit large amplitudes across a range of periods, as expected from Fourier theory. If the broad-spectrum behavior of the input data is relatively time-localized, such as the runoff response to a brief storm event, the scalogram will exhibit lineations corresponding to that event. Thus, temporally continuous but spectrally limited high amplitude regions indicate periodic processes within the input data while temporally limited but spectrally broad lineations indicate broad-spectrum processes.

**SCALOGRAM AVERAGING**

Averaging the scalogram amplitudes across periods or time gives the period-averaged or time-averaged amplitude spectra, respectively, of the SWFT scalogram [Torrence and Campo, 1998]. All of these three averages are computed using a rectangularly interpolated grid, $A_{sr}$ at user-selected periods $s$ and times $r$, from $A_{pq}$. Since the period spacing in the SWFT scalogram is not necessarily uniform, we use the period-weighted mean amplitude $Ap$, calculated using

$$Ap = \frac{\sum_{s=1}^{S} P_s A_{sr}}{\sum_{s=1}^{S} P_s}$$

(17)
where $L$ is the dimension of the rectangular grid in the period direction. This provides aggregate information on the temporal variation of spectral amplitude.

The time-averaged amplitude $A_t$, given by

$$A_t = \frac{1}{M} \sum_{r=1}^{M} A_r$$

(18)

where $M$ is the dimension of $A$ in the time direction, displays average amplitudes across time at a single period. Referred to as the global SWFT spectrum, this time-averaged spectrum is qualitatively similar to the FT spectrum, but differs in physical meaning.

Bulk changes in the relative influence of short- vs. long-period fluctuations across time can be visualized using the amplitude-weighted average period $P_r$,

$$P_r = \frac{\sum_{r=1}^{L} P_r A_r}{\sum_{r=1}^{L} A_r}$$

(19)

All three of these scalogram averages are demonstrated below.

**WATERSHED AVAILABLE PRECIPITATION: SNOWMELT MODELING**

Although this study focuses on revealing and exploring watershed process using a purely data-driven SA, one model is required for the analyses. Because precipitation falls as snow during most of the winter months in northern lower-Michigan, a snow storage-and-release model is needed. The data required for a full energy/water balance model were either unavailable or incomplete, thus we implemented a simple heuristic snowmelt model. Because data are available for both fresh snow totals and observed snow depth, the model must simply identify when a decrease in snow pack thickness corresponds to a melt event or densification of the snow pack. This heuristic model tracks snow water equivalent and releases snowmelt based on a three-part rule structure:

1) The density of newly-fallen snow is calculated from daily precipitation and snow fall totals. Since precipitation can be mixed frozen and liquid, a maximum new-snow density cutoff, $new_{max}$, was determined from the data. Any precipitation in excess of this cutoff is considered equivalent to rainfall and immediately released.

2) The total snowpack density is updated based on the new snow depth and the accumulated water content of the pack.

3) If the average snowpack density exceeds the maximum pack density, $pack_{max}$, it is assumed that some of the snow has melted. Melt is then generated equivalent to the depth of the pack multiplied by the difference between the calculated pack density and the maximum density.

There are three important assumptions in this model. First, we assume that drifting does not affect the observed snow depths at the measurement location (typical of snow models). Second, we assume that the snow pack properties are uniform throughout, which is realistic in the MRW region as total pack thicknesses are typically less than a third of a meter. Finally, this model assumes that the density at which the snow pack releases water remains the same throughout the season.

The maximum densities of the new snowfall and the snow-pack are physical quantities that are generally not equal. Maximum snowpack density, $pack_{max}$, is determined by both its composition and water holding capacity, which are functions of the thermal history of the pack and new snowfall conditions. Direct comparison of snow depth and stream discharge in our study area suggests that water is released from the snowpack when the combined snow/water density reaches approximately 0.35 g/cm$^3$, according to this model, thus this value was chosen for $pack_{max}$. The maximum new snowfall, $new_{max}$ density of 0.23 g/cm$^3$ (determined as the ratio of new snow water content to new snow depth) was extracted from the data, as this was the relatively abrupt limit above which higher-density new snowfall events were obvious outliers.

To assure that the heuristic model performs acceptably, it was compared to the UEB snow model [Tarboton and Luce, 1996] for the winter of 1999/2000. The solid line in Figure 3a is the heuristic snowmelt model, and the dashed line is the output from the UEB snow model. Both models output the combined snowmelt and liquid precipitation, referred to hereafter as watershed-available precipitation. Watershed-available precipitation is that which can be acted upon by physical or biological processes. Note that the models provide very similar results, with the UEB model predicting slightly more melt early in the season and the heuristic model predicting more melt late in the season. The heuristic model predicts approximately 1.5 cm more snowmelt during the season than the UEB model. However, the heuristic model does not allow for sublimation, which entirely accounts for the ~1.5 cm difference at the end of the modeled period. The heuristic model was chosen for our analysis since it performs acceptably, despite minimal data requirements and a simple structure.
Examining Watershed Processes: Spectral Comparison

The spectra of stream discharge, watershed-available precipitation, and relaxation of the water table elevation in well B13 (Figure 1) can be directly compared to infer interaction timescales between the data types and to examine details of processes within each type. Well B13 (data not plotted) was chosen because the water table is deep enough (>30 meters) to preclude direct evapotranspiration effects. Four key features of the spectra will be compared to integrate the hydrologic datasets and reveal process details: 1) spectral peaks, 2) log-log linear slopes (i.e., fractal scaling), 3) locations of slope-breaks, and 4) relative spectral amplitudes. Since watershed-available precipitation is the primary forcing function for natural watershed processes in our study region, the spectra of well-head relaxation and stream discharge can be viewed as modifications, or fractal filters [Kirchner et al., 2000], of the watershed-available precipitation spectrum. The same is true to some degree for the well-head relaxation and stream discharge spectra, as groundwater inputs to the Muskegon River account for a majority of its annual discharge [Jayawickreme and Hyndman, 2007].

The spectra of these three data types are plotted in Figure 4 along with log-log linear fits and 95% confidence intervals. Also plotted is the fixed period-width binned spectrum to aid visualization. Slopes for selected linear portions of the spectra were calculated using least-squares regression between user selected bounds. These bounds were chosen to match portions of the spectrum that exhibited a linear slope. The slope breakpoints are then calculated at the intercepts of the separate linear fits.

The 95% confidence interval (dotted line) is determined by multiplying the $\chi^2$-square value for a system with 1 degree of freedom by the average amplitude given by a noise (or scaling) model [Torrence and Compo, 1997]. In this case, the noise model was assumed to be given by the linear fits. This enables the flexibility of applying statistical confidence intervals to datasets without assuming a priori a particular type of noise. This is useful when working with spectra that exhibit multi-scaling behavior [Tessier et al., 1996; Dahlstedt and Jensen, 2004], where a single-scaling noise model, and therefore confidence test, would be inadequate.

All three process spectra have annual-cycle peaks at or near 365 days, although the peak in the head relaxation data is significantly below the 95% CI boundary. This is probably due to the relatively short data record available for the head relaxation. The discharge spectrum has a series of peaks at the harmonics of the 1 day peak that are artifacts, as later discussion will demonstrate. The head relaxation spectrum also has a peak near 117 days, along with weaker peaks near 70 and 50 days (Figure 4d). These are near the integer factors 3, 5, and 7 of the 365 day peak. The watershed-available precipitation spectrum has two additional peaks at ~174 days, and another at ~65 days. These may not indicate sinusoidal processes active at those periods, but may instead be the spectral signature of a non-sinusoidal process characterized primarily by a longer-period oscillation. In particular these two peaks are near the integer factors 2 and 6 of the primary 365 day peak.

The slopes of the spectra and slope breaks (Table 2) provide provocative evidence of linkages between watershed processes. If a quantity, such as stream discharge, is being forced directly by another, such as precipitation, then fractal scaling active in the forcing input should exhibit itself directly in the response variable [Tessier et al., 1996]. However, a non-linear system response behaves like a fractal filter, modifying the input scaling behavior [Kirchner et al., 2000]. There are two examples of this in Figure 4 and Table 2. The first is the segment of the discharge spectrum between one and three hours with a $\beta$ of 1.5. This is roughly similar to the slope of the NEXRAD hourly precipitation spectrum. The $\beta=0.9$ slope in the NEXRAD spectrum is an underestimate of the true spectral slope, given that the hourly NEXRAD spectrum represents just a single year of
data that undersamples precipitation, and thus suffers from high-frequency aliasing and slope-flattening as described in Kirchner [2005]. If, as indicated in these empirical spectra, the “true” spectral slopes of streamflow and precipitation match in this range, we interpret the similarities of slope up to a period of three hours to indicate that direct precipitation is the dominant flow-generation process. Beyond three hours, processes with a different scaling relationship dominate flow.

Process linkages are also apparent between long-period stream discharge and head relaxation spectra. Both spectra have a slope break at approximately 16 days and follow a $\beta=1.1–1.2$ scaling relationship to longer periods. This suggests that variations in groundwater discharge control stream discharge variability at periods longer than approximately 16 days. The exact value of this slope break is approximate because of the gradual transition between linear segments in the discharge spectrum between approximately 10 and 30 days. Zhang and Schilling [2004] observed slope breaks in Iowa streams at approximately 30 days. The similarity in slopes between head-relaxation and discharge in Figure 4 suggests that groundwater inputs dominate streamflow in these Midwestern streams for periods longer than 10–30 days while in-stream and near-surface watershed processes appear to dominate at shorter periods.

The spectral slope values also provide useful information. Particularly interesting is the $\beta=2.9$ slope seen at periods between about 3 hours and 16 days in the discharge spectrum. The uniformity of scaling in this portion of the spectrum means that any watershed processes active in this period range also exhibits similar scaling. Fundamentally, this is because linear processes that are additive in the time-domain also add in the spectral domain [Smith, 1997]. If the scaling of any hydrologically significant watershed process differed from the others, it would preclude the observed uniform scaling. This uniformity is interesting considering the variety of watershed processes active in this period range, including bank storage and release, precipitation runoff, near-surface “interflow”, near-stream saturated groundwater response, and evapotranspiration. Further investigation of this uniformity could be undertaken with a more concentrated set of data designed to explore these processes, or using a detailed process-based hydrological model.

**Figure 4.** Composite plot of stream discharge, well-head relaxation, and precipitation spectra. Grey lines plot the raw FT spectra, solid lines are the binned-mean amplitudes, and dashed lines give the upper 95% confidence intervals. a) An overlay of the spectra in parts c–e and their linear fits; b) NEXRAD precipitation spectrum for 4/1/2004–11/30/2004, not plotted on part a; c) stream discharge spectra for 10/1/1997–9/30/2004; d) well-head relaxation for well B13 from 7/1/2003–7/1/2005; and e) watershed-available precipitation from 1/1/1990–1/1/2001. Note that, in part c the limited resolution makes it appear as if much of the stream discharge spectrum is above the confidence limit, however fewer than 5% of points exceed the limit in a spectrum with over 130,000 points.
Analysis of the relative amplitudes of the three spectra in Figure 4a provides a hydrologic response time to precipitation. The amplitudes of watershed-available precipitation and discharge converge at a period of approximately 2.5 years. Thus any long-period fluctuation in precipitation will be followed by an equal magnitude fluctuation in stream discharge. Therefore this watershed and its unsaturated and saturated groundwater processes do not appear to control hydrologic fluxes with periodicities greater than 2.5 years. This observation could be used in autoregressive models of basins with short discharge records but more extensive precipitation data. Another interpretation of the similarity in magnitude between the precipitation and discharge spectra is that the combined response time of both surface and groundwater systems is approximately 2.5 years under current conditions, although extended drought periods could certainly affect this value. Therefore, in order to assure insensitivity to initial conditions, a model of this watershed must be “spun up” with realistic meteorological inputs for at least 2.5 years.

Beyond 10–30 days, the well-head relaxation amplitudes are approximately 2–3 times greater than those of stream discharge. This disparity is likely a combination of both physical processes as well as our assumptions. First, the well-head relaxation amplitudes would exceed those of stream discharge because of evapotranspirative losses. Also, the head in an individual well may fluctuate more than the average of all wells in the watershed, particularly if the water table at that well is deep. Thus a more representative comparison would be between stream discharge and the average of spectra from wells distributed across the watershed. However, in this case many of our wells showed signs of periodic anthropogenic disturbance that violate assumptions of our simple differencing technique. Other factors that contribute to the disparity between head relaxation and stream discharge amplitudes may include overestimation of porosity or average recharge rate because of the short data record in the well, as well as differences in annual recharge between the Evart watershed and the location of the well in the Grand Traverse Bay region.

The head relaxation results presented in Table 2 are taken from the spectrum of smoothed-differentiated head fluctuations shown in Figure 4d while the head fluctuation data in Table 2 were taken from the spectrum of actual head fluctuations (not plotted). Unlike head fluctuations, head relaxation (and therefore a decrease in storage) is directly related to groundwater discharge, allowing direct comparison of their spectra. Other studies have reported head fluctuation spectra [Zhang and Schilling, 2004; Lee and Lee, 2000; Naff and Gutjahr, 1983], but because the basic units of head fluctuation and stream discharge differed in these studies, their reported groundwater head and baseflow scaling laws can not directly be related.

### EXAMINING INTRA-ANNUAL SPECTRAL VARIABILITY USING THE SWFT

Physical interpretations of Fourier Transform spectra assume that system processes are both non-intermittent and stationary. However, many watershed processes are either inactive for parts of the year (intermittent) or possess different spectral characteristics over successive cycles (non-stationary). Thus, interpreting spectra from many occurrences of a given process is problematic. To overcome this, we use the Scaled-Windowed Fourier Transform (SWFT), which does not require either stationarity or non-intermittency. As Figures 5–10 demonstrate, all three watershed processes examined in this study exhibit both non-stationary and intermittent processes.

The SWFT spectrum of stream discharge (Figure 5) displays its time-varying Fourier spectral content, revealing details about how discharge responds to hydrologic inputs and watershed processes. As described previously, the vertical lineations in the SWFT scalo gram are caused by the broad spectrum of time-series stream discharge peaks with each such lineation corresponding to a pulse increase in stream discharge. Clearly separated lineations, which primarily occur during the summer months, extend from very short periods and tend to reach maximum amplitudes at periods between 20–40 days. This range of periods cor-

<table>
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<th>Watershed-available Precip</th>
<th>Head Relaxation</th>
<th>Head Fluctuation</th>
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<td>0.90</td>
<td>0.08</td>
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</tbody>
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1 These bold italicized entries correspond to results from the NEXRAD dataset, normal entries refer to the watershed-available precipitation from gage data.
2 These italicized entries correspond to slopes from Well B13, whose binned-mean spectrum is not plotted.
responds to the width of the time-series discharge peak, and thus to the time-scale of surface and near-surface watershed response to precipitation inputs. As will be shown in the next section, this 20–40 day time scale of surface and near-surface hydrologic response is also similar to the width of the unit hydrographs developed for this watershed.

Spring months typically exhibit increased Fourier amplitudes relative to other seasons across all periods shorter than several hundred days. The spring of 1998 has a very different spectrum than that of 1996 or 1997. Those years had several spring discharge peaks followed by relatively high summer baseflow levels, whereas the single discharge peak of 1998 is followed by low baseflow and weakening of the 365 day amplitude. This summer baseflow portion of the time-series discharge is accompanied by decreased amplitudes across nearly all periods, except near 365 days. Thus, stream discharge during this portion of the year is dominated by long-period fluctuation, most likely from groundwater inputs, as indicated by the amplitude-weighted mean period curve ($P_t$).

Seasonal differences in spectral character between the summer and spring/fall are not evident in the watershed-available precipitation SWFT scalogram (Figure 6c), but show up very prominently in discharge (Figure 5c). This suggests that although there is spectral power in that period range in watershed-available precipitation, summer evapotranspiration and canopy interception decrease the magnitude of discharge response and thus the spectral amplitudes.

Another important difference between Figures 5c and 6c is that the dominant power in the precipitation spectrum occurs at shorter periods while the reverse is true for stream discharge. This corresponds to the behavior seen in Figure 4c, however the time-averaged amplitude ($A_t$) spectrum of watershed-available precipitation differs from the Fourier spectrum in Figure 4e. Such differences are artifacts produced by the violations of the assumptions of stationarity and non-intermittency inherent in the standard FT.

![Figure 5](image.png)

**Figure 5.** SWFT of stream discharge at the USGS gage in Evart, MI. a) Stream discharge time-series divided by the area of the watershed above this gage; b) Scale-averaged amplitudes, $A_p$; c) Filled contour scalogram of the amplitude ($A$) vs. both period ($P$) and time ($t$), along with the location of the amplitude-weighted mean period, $P_t$ (white line); d) Time-averaged amplitude spectrum, $A_t$. 
The SWFT discharge scalogram for the water years 10/1/1991–9/30/2004 (Figure 7) displays the spectral content of stream discharge at the Evart gage over periods of 0.3–1000 days. Although the longer-period spectrum is not plotted, 2–3 year and 6–8 year cycles are evident in periods between 100–400 days, perhaps related to climate cycles as seen in Coulibaly and Burn [2004]. For 1991–93, long-period fluctuations are most active near a period of 180 days, which then cease during 1994–95 before resuming from 1996–98 at 365 days. Again, this long-period activity switches off for two years and then resumes centered about 180 days. Another interesting set of features in this scalogram are the summer-fall periods of 1998, 2000, and 2002 in which amplitudes are drastically reduced up to periods on the order of 100–200 days.

These observations from Figure 7 enable a deeper understanding of time-averaged spectra such as the FT spectrum or the SWFT time-averaged amplitude ($A_t$). The FT spectrum of discharge (Figure 4c) contains a weak spectral peak at 180 days along with a peak at 365 days. Prior to examining the spectrum as a scalogram, we were unable to distinguish between dominant spectral peak harmonics and peaks from independent processes at those periods. The scalogram shows that there are processes generating true peaks at both the 365-day and 180-day periods.

Additionally, we can use the information from the scalogram to indicate when processes that generate particular spectral peaks are most active. A prominent example of this is the 1 day peak in the FT spectrum of discharge. Plausible interpretations of this 1 day peak include diurnal fluctuation in evapotranspiration or streambed conductance during the summer months. However, Figure 7 shows that the dominant 1 day amplitudes occur in the winter and early spring months. A close examination of the time-series reveals two important details: 1) the diurnal fluctuation is strongest during periods where daily maximum temperatures are subfreezing, and 2) discharge peaks during the coldest mid-morning hours of each day. These observations suggest that the diurnal signal may be related to icing effects at the instrument. The 1-day peak seen in Figure 4 is a “true” spectral peak, but it does not

Figure 6. SWFT of watershed-available precipitation at Big Rapids, MI. a) watershed-available precipitation time-series; b) Scale-averaged amplitudes, $A_p$; c) Filled contour scalogram of the amplitudes ($A$) vs. both period ($P$) and time ($t$), along with the location of the amplitude-weighted mean period, $P_t$ (white line), (note, white areas have amplitudes $< \log(\text{amplitude})=-3.5$); d) Time-averaged amplitude spectrum, $A_t$. 
indicate cyclic system processes so much as inaccuracies in measurements. Also, there are no significant amplitude peaks at the 0.5-day period, thus confirming that the corresponding peak in Figure 4c was indeed a harmonic. This information could then be used to filter the discharge time-series and remove this apparently erroneous periodic discharge behavior.

The SWFT of head fluctuations from Well B10 (Figure 8) provides another example of the importance of viewing spectra as a function of both period and time. Well B10 was chosen because it exhibited the greatest amplitude of fluctuation in the period 1–30 days of the 17 wells in our study. Figure 8 displays both the head fluctuation data (mean removed) as well as the SWFT scalogram for periods between 0.3 and 100 days. The largest time-series amplitude head fluctuations occur from late October to early April. The scalogram reveals that most of the time series fluctuation is caused by larger Fourier amplitudes of the 1–30 day periods, which are greatest during the winter. Note that the dominant period in this range is not constant throughout the year. During the summer, early fall, and spring, the dominant period is near the 7–10 day range, that then shifts to the shorter 2–7 day range in mid-November. The SWFT scalogram clearly reveals this information that would be difficult to directly extract from the time-series data.

SPECTRALLY DERIVED WATERSHED “UNIT HYDROGRAPH” RESPONSE FUNCTIONS

The watershed annual unit response functions for the portion of the Muskegon River Watershed above Evart gauge were calculated using discharge and Big Rapids precipitation data between 9/1/1999 and 8/31/2000 (chosen arbitrarily). Two different unit response functions are shown in Figure 9a, that of the discharge response to watershed-available precipitation as well as to raw precipitation. Including the snow storage-and-release model creates higher peak discharge responses with a more physically realistic long tail due to groundwater discharge. The higher peak is expected because without a snow model this method treats the precipitation the same in January as it would in July, even though the January precipitation fell as snow and was stored until later, resulting in no significant short-term discharge response.

A convolution of the solid-line unit response function in 9a with watershed-available precipitation according to Equation 1, produces the dashed modeled discharge and residual curves in Figures 10a and 10b. Because the unit-response was truncated as described in the methods section, the convolution is not a perfect reconstruction. The resultant discharge is an overestimate in the summer and fall months, but an underestimate during the spring. This is to be expected as the annually calculated response curve effectively averages the system behavior throughout the year.
If the unit response is calculated seasonally rather than annually, the resultant set of unit response curves reveals the seasonal differences among runoff responses in this watershed (Figure 9b). Discharge response during the spring is much higher than either the annual curve or those of the other three seasons, perhaps due to a combination of frozen soils and higher average soil saturation prior to watershed-available precipitation events (which can be either rainfall or snowmelt). Summer discharge response, on the other hand, is highly damped due to canopy interception, lower average soil moisture, and evapotranspiration. The fall and winter responses appear to be very similar, suggesting that there are not large differences in discharge response between these two seasons. This result was somewhat unexpected since frozen soils are generally expected during the winter months due to long periods of sub-freezing temperatures. Significant areas of frozen soils would tend to increase discharge response, as infiltration capacity is greatly reduced. The lack of a response difference suggests that the soils are not homogeneously frozen throughout the winter season, or that this freezing is not important for runoff generation in this watershed during this time period.

The response curves in Figure 9b all converge to near 0 beyond approximately 20 days. Analysis of data from the wells in the GTBW indicates that the delays between peak spring recharge and peak saturated water table response scales approximately as 2–4 days/meter of unsaturated zone depth. Thus, for depths on the order of 30 meters, the delay between full groundwater response to a precipitation (or snowmelt) event can be as much as 120 days. As the data lengths included in the seasonal response calculations are only on the order of 60–90 days the seasonal unit response curves can not properly represent the groundwater response. The tail in the annually calculated unit response curve of Figure 9a is likely a more reasonable representation of the average groundwater response.

In addition to providing more insight into watershed processes than the annually-derived unit response curve, the seasonally-derived curves produce a much more accurate estimate of stream discharge when convolved with watershed-available precipitation (Figure 10a). The residuals between the two convolutions and stream discharge (Figure 10b) quantitatively demonstrate the improvement in discharge estimation gained by seasonal convolution and consideration of non-stationary system behaviors. Figure 10a illustrates the utility of the seasonally-derived watershed unit response curves for providing a very simple means of forecasting discharge response to precipitation events. Because the seasonal curves average watershed responses across significant variability in watershed state properties (such as soil moisture), the seasonally-derived convolution underestimates the largest peaks in the discharge data by up to 50%, while predicting measured flows to an accuracy of +/- 25% in most other cases. Much of the remaining residual is because the 50-day unit response curves fail to capture
much of the groundwater response to spring and fall precipitation. An analysis that accounts for the different seasonal responses between wet and dry years, and explicitly incorporates the full groundwater response, might further improve the accuracy of forecasting with this technique.

CONCLUSIONS

We present the application of three spectral analysis techniques, direct spectral comparison, the Scaled Windowed Fourier Transform (SWFT), and the derivation of the unit hydrograph via FT deconvolution. We have discussed and demonstrated how each technique requires consideration of limitations and possible pitfalls in order to be applied successfully, and we elucidated a set of best practices in their application. Most importantly, we have demonstrated that these spectral analysis methods can be used to integrate hydrologic data in order to evaluate watershed processes.

The spectra of related data types were directly compared to infer process linkages between hydrologic inputs, watershed processes, and stream discharge. Similarities in amplitude peaks, log-log linear fractal scaling behaviors, and breaks in scaling slopes among datasets indicate the nature of linkages. For example, fractal scaling in precipitation may be matched in the stream spectrum at periods shorter than approximately 3 hours for the Evart, MI sub-basin of the Muskegon River Watershed. From periods of 3 hours to approximately 10-30 days, stream scaling follows a $\beta=2.9$ slope. This single scaling relationship is notable considering the variety of processes active in this period range, suggesting mathematical similarities among these processes. Beyond 30 days, the scaling apparent in the stream spectrum appears to be controlled by groundwater inputs. But, past a period of approximately 2.5 years, fluctuations in the precipitation spectrum control the stream discharge spectrum. This overall watershed response time should be considered when developing transient predictive simulations of watershed behavior.

We introduced the Scaled Windowed Fourier Transform (SWFT) technique to examine the time-varying content of fundamentally non-stationary hydrologic datasets. The SWFT scalograms revealed both the non-stationarity and intermittency of stream discharge, precipitation, and groundwater head fluctuation spectra. The effect of evapotranspiration and canopy interception is evident in a comparison of the SWFT scalogram of summer precipitation events to the highly damped discharge scalogram for those seasons. Also, the 1-day peak evident in the FT spectrum of stream discharge was shown to be due largely to measurement error rather than diurnal hydrologic processes. Importantly, these are a subset of many possible observations from the rich set of information contained within the SWFT scalogram.

Using direct FT deconvolution, spectral analysis can be also be used to estimate stream unit hydrographs. Because of the simplicity of this method, temporally- and seasonally-varying hydrographs can be quickly derived to better understand non-stationary watershed processes. For the Muskegon River above Evart, MI, the groundwater dominance of the stream discharge spectrum beyond approximately 15–20 days is confirmed by visual inspection of the main unit response peaks. These peaks show a 15–20 day primary stream response period followed by a long-tailed groundwater response that continues out to at least 50 days. Also, the seasonally-derived unit hydrographs quantitatively reveal decreased discharge responses due to evapotranspiration during the summer months and augmented responses during spring snowmelt.

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