Traveltime inversion for the geometry of aquifer lithologies

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ABSTRACT

Crosswell traveltime tomography can provide detailed descriptions of the geometry and seismic slowness of lithologic zones in aquifers and reservoirs. Traditional tomographic inversions that estimate a smooth slowness field to match traveltime data, provide limited information about the dominant scale of subsurface heterogeneity. We demonstrate an alternative method, called the multiple population inversion (MPI), that co-inverts traveltimes between multiple well pairs to identify the spatial distribution of a small number of slowness populations. We also compare the MPI with the split inversion method (SIM) that was recently introduced to address the same problem. The lithologies and hydraulic parameters for these populations can then be determined from core data and hydraulic testing.

The MPI iteratively assigns pixels to a small number of slowness populations based on the histogram of slowness residuals. By constraining the number of slowness values, this method is less susceptible to inversion artifacts, such as those related to slight variations in ray coverage, and can resolve finer scale sedimentary structures better than methods that smooth the slowness field. We demonstrate the MPI in two dimensions with a synthetic aquifer and in three dimensions with the Kesterson aquifer in the central valley of California. In both cases, the constrained inversion algorithm converges to an equal or smaller average traveltime residual than obtained with unconstrained-value tomography. The MPI accurately images the dominant lithologies of the synthetic aquifer and provides a geologically reasonable image of the Kesterson aquifer.

INTRODUCTION

Many geoscience and engineering disciplines would benefit from detailed descriptions of the geometry of lithologic zones. Crosswell seismic tomography, which bridges the resolution gap between surface seismic and log measurements, has the potential to provide such descriptions. Crosswell methods may be used to characterize contaminated groundwater sites, but to date they have been used primarily to characterize petroleum reservoirs (Lendzionowski et al., 1990; Harris et al., 1995).

The ultimate goal of both petroleum and environmental site characterization is to estimate the spatial distribution of lithologies and their associated fluid-flow properties. However, the smoothed slowness estimates from unconstrained-value tomography may not identify distinct lithologies adequately. Since slowness values for lithologies tend to cluster, the predominant lithologies in a region may be identified more accurately using an approach that constrains the number of slowness values rather than spatially smoothing the estimated slowness field. The trade off for improved spatial resolution of lithologies will likely be reduced resolution of slowness variability. In addition, hydraulic interpretation of seismic tomograms is difficult because the relation between seismic and hydraulic properties is usually unknown and probably nonunique. Although seismic slowness values may be correlated poorly to hydraulic property values, significant slowness contrasts often relate to changes in lithology. Thus, the geometry of the predominant lithologies can be estimated using seismic traveltimes, and then hydraulic properties can be estimated from core samples, geophysical logs, and hydraulic and tracer tests.

Some work has been published on methods to estimate the zonation of lithologies based on seismic data. Lendzionowski et al. (1990) found that pattern recognition methods, such as projection pursuit, can significantly aid the interpretation of seismic reflection data. Carrion (1991) used dual tomography to constrain seismic velocity values, which improved the estimates for a zonal synthetic model. Hyndman et al. (1994) presented the split inversion method (SIM) to co-invert seismic traveltimes and tracer (i.e., fluorescent dye) concentrations for
a small number of lithologic zones and the seismic and hydraulic parameters for each zone. Synthetic examples were used to demonstrate the SIM for a case in which the relation between seismic slowness and hydraulic conductivity was non-unique and unknown. By combining seismic and tracer data, the SIM was able to estimate both seismic slowness and hydraulic conductivity values without a specified relation between these parameters. This is a critical element of any method that combines seismic and hydraulic data because there is an inherent non-uniqueness in the relation between parameters of wave propagation and parameters of fluid flow and solute transport (Han et al., 1986; Marion et al., 1992).

In this paper, we develop the multiple population inversion (MPI) to estimate the geometry of dominant seismic slowness populations in a region using traveltimes alone, rather than the SIM approach of splitting an unconstrained-value tomogram into zones. We implement the MPI with both synthetic and field P-wave traveltimes. The method could also be modified to use other types of data that provide detailed spatial information about lithologic contrasts.

The objective of the MPI is to minimize the average absolute traveltimes for rays simulated through pixels constrained to a small number of seismic slowness values. By limiting the number of possible slowness values, the MPI efficiently and effectively estimates the geometry of the predominant lithologies between two wells. In regions with multiple cross-sections or multiple wells, the MPI can coinvert all available traveltimes to generate consistent regional slowness estimates. These regional estimates can then be interpolated into three dimensions and used in flow-property inversions.

**ZONAL INVERSION METHODS**

We have developed two zonal inversion methods, the split inversion method (SIM) and the multiple population inversion (MPI), to determine the geometry of dominant lithologic zones and parameter values for these zones. Both methods are capable of estimating the structure of dominant slowness populations from seismic traveltimes, and then estimating lithologies and hydraulic properties for each population using information such as well logs. The assumption, hence the philosophy, behind these inversions is that information about the zonation of lithologies is contained in crosswell seismic traveltimes.

**Split inversion method (SIM)**

The SIM, which was introduced in Hyndman et al. (1994), assumes that an unconstrained-value seismic tomogram contains accurate information about slowness structure and thus splits this tomogram into zones using a post-inversion step. The objective, using seismic data alone, is to find the seismic slowness zonation that minimizes traveltimes residuals. For a two population case, the zonation is adjusted by changing the estimated seismic slowness value that separates low slowness zones from high slowness zones. For each evaluated zonation, a slowness value is assigned to each zone based on the histogram of slowness values in the unconstrained-value tomogram. In other words, the SIM assumes that information about the slowness value for each population can be derived from the histogram of slowness estimates. However, this assumption could be relaxed by directly inverting the matrix of traveltime equations for the zonal slowness values using the path lengths through each zone, which would provide the best slowness values for each potential zonation.

The authors also demonstrated the SIM for the co-inversion of seismic and tracer data to estimate the best lithologic zonation and hydraulic and seismic slowness for each zone. In this manner, lithologic information can be derived from multiple complementary data sets. The SIM is thus a flexible inversion method that can be formulated to best combine multiple data types.

**Multiple population inversion (MPI)**

**Overview.**—The MPI method is designed to co-invert seismic traveltimes taken between several well pairs for the regional zonation of seismic slowness. These slowness zones can then be interpreted as lithologies, assuming that a connected region of similar slowness represents a lithology. While the relationship between slowness and lithology may not be one-to-one and certainly not unique, subsequent interpretation of the MPI estimate may be useful in delineating zones with similar properties, such as clay content, porosity, or even permeability.

The MPI estimates the spatial distribution of a small number of slowness populations. Thus, this approach parameterizes the inversion to estimate zones rather than extracting zonal information from a previously estimated tomogram, as is done for the SIM. The MPI is implemented in a similar manner to unconstrained-value tomography, but residual artifacts are reduced greatly in the MPI by constraining the number of slowness values acceptable in a region. This approach infers lithologic structure without the potential biasing effect of smoothing, which is a common method of reducing artifacts in tomographic inversions. In cases where multiple scales of heterogeneity are expected, the final multipopulation estimates can be used as starting models for unconstrained-value tomographic inversions.

The MPI was developed using crosswell seismic traveltimes because path integral inversions provide high resolution information about slowness contrasts between wells. These traveltimes are used in an iterative inversion that updates slowness estimates to minimize traveltime residuals. The foundation for this implementation of the MPI is a small perturbation expansion of the traveltime equation (1), which we derive below.

**Mathematical development.**—For each iteration of the MPI, we simulate (M) raypaths through the latest estimate of the slowness field using a fourth order Runge-Kutta approach. We approximate the traveltime ($t_i$) for the $i$th source-receiver pair as the integral of slowness ($S$) along the simulated raypath ($C_i$):

$$t_i = \int_{C_i} S(x, z) \, d\ell \quad i = 1, \ldots, M \text{ rays} \tag{1}$$

where ($d\ell$) is the increment of path length along a ray.

We use ($N$) rectangular pixels as orthogonal basis functions ($\psi_j$) to provide a discrete representation of slowness:

$$S(x, z) = \sum_{j=1}^{N} S_j \psi_j(x, z) \tag{2}$$
where the basis functions

\[ \psi_j = \begin{cases} 1 & \text{inside } j\text{th rectangular pixel} \\ 0 & \text{outside } j\text{th rectangular pixel} \end{cases} \]  

\[ t_i = \sum_{j=1}^{N} S_j \xi_{ij} \quad i = 1, \ldots, M \text{ rays}, \]  

where

\[ \xi_{ij} = \int_{C_{ij}} \psi_j(x, z) d\ell \]  

Both the raypath and the slowness field are unknown in equation (3), thus we cannot invert this equation directly for the unknown slowness field. Instead, we linearize equation (3) using a small perturbation approach. We write the slowness field \( S_j \) as the sum of a known background field \( S_{0j} \) plus a field of unknown slowness perturbations \( \Delta S_j \) as follows:

\[ S_j = S_{0j} + \Delta S_j \quad j = 1, \ldots, N \text{ pixels}. \]  

We replace the unknown raypath in equation (1) with the raypath \( (C_{0i}) \) simulated through the known background field \( (S_{0j}) \), which is the slowness estimate from the previous iteration. Thus, equation (3) reduces to a linear set of equations (5) that relate traveltime residuals \( (\Delta t_i) \) to slowness perturbations \( (\Delta S_j) \) using a projection matrix \( (\xi_{ij}) \), which has elements of raypath length in each pixel.

\[ \Delta t_i = \sum_{j=1}^{N} \Delta S_j \xi_{ij} \quad i = 1, \ldots, M \text{ rays}. \]  

We then calculate the slowness perturbations \( (\Delta S_j) \) by inverting equation (5a) using the simultaneous iterative reconstruction technique (SIRT) in Harris et al. (1990). Other inversion schemes, such as conjugate gradients, could also be used instead of SIRT.

Rather than adding the spatial field of slowness perturbations to the background slowness field, as would be done in most tomographic inversions, the MPI separates the slowness field into a small number of populations using the histogram of the estimated slowness perturbations \( (\Delta S_j) \).

**Inversion steps.**—The following steps are used for each MPI iteration (Figure 1):

1. Simulate raypaths through the known background slowness field and calculate traveltime residuals.
2. Invert equation (5a) for a spatial field of slowness perturbations using SIRT.
3. Assign pixels to a small number of slowness populations based on the histogram of values in this spatial field of slowness perturbations.
4. Calculate the representative slowness value for each population, as the previous estimate plus the median of the slowness perturbations for pixels assigned to a population.
5. Check that each pixel’s population change does not result in an unreasonable slowness change. Place all pixels that experience extreme slowness changes [i.e., greater than ten times the estimated slowness perturbation from step (2)] back to their previous population. This allows for smooth convergence of the traveltime residuals.

The MPI is an iterative estimation algorithm, thus an initial guess of the seismic slowness field is needed. A homogeneous background slowness field is an unbiased choice, although prior geologic information could be used to construct an initial multiple population slowness model. The Householder transform (Golub, 1989) was used to invert the overdetermined system of traveltime equations for a regional value of slowness \( (S_0) \), although other methods such as least-squares inversion also could be used.

The number of slowness populations allowed can be set equal to the number of predominant lithologies in the region, based on core samples and geologic inference. A specified proportion of pixels with the largest absolute slowness perturbations are then assigned to populations based on the histogram of slowness perturbations (Figure 1). This proportion does not include pixels that belong to the extreme population in the direction of the calculated slowness perturbation. This allows pixels to change populations for later MPI iterations when most of the pixels are constrained to a population. In practice, the population assignment is done by ranking the absolute slowness perturbations for all pixels in the region and calculating the cutoff perturbation value that allows the desired proportion of changes. Pixels are only allowed to change if the absolute perturbation is greater than this determined cutoff value, and pixels can only change one population per iteration.

Our experience demonstrates that populations should be progressively added to obtain the expected number of populations, rather than starting the inversion with multiple populations. Each population is added in the direction of the skew of the slowness perturbation histogram (Figure 1). Thus, if this histogram is skewed toward large negative slowness perturbations, a low slowness population would be added. If no prior information exists, the number of populations can be increased until the average traveltime residual is no longer reduced by repeated iterations and the slowness perturbation histogram has little skew.

Each population \( (p) \) is then assigned a representative slowness value \( (S^p) \), which is the population’s previous slowness estimate \( (S^p_0) \) plus a perturbation \( (\Delta S^p) \). Thus equation (4) has been modified to constrain the slowness values to \( (K) \) populations and obtain

\[ S^p = S^p_0 + \Delta S^p \quad p = 1, \ldots, K \text{ populations}, \]  

where

\[ \Delta S^p = \text{median}(\Delta S^p_j) \quad j = 1, \ldots, N \text{ pixel}. \]  

In this implementation of the MPI, each perturbation \( (\Delta S^p) \) is calculated as the median of the slowness perturbations for pixels assigned to a population. Thus, each population is parameterized by a single slowness value. The median of the perturbations is used because it is less sensitive than the mean to the largest perturbations.
The objective of our inversion is to minimize the average absolute travelt ime residual as follows:

\[
\text{Minimize } \left\{ \left( \sum_{i=1}^{M} |\Delta t_i| \right) / M \right\} .
\]  

(7)

For each MPI iteration, rays are traced through the perturbed slowness field and the changes are accepted as long as the value of the objective function, called the objective value, is reduced or at worst slightly increased. This is similar in philosophy to simulated or threshold annealing (Dueck and Scheuer, 1990).

Annealing algorithms allow large numbers of changes to an estimated field early in the inversion when the estimate is far from the optimal value. Fewer changes occur with later iterations as the estimate converges toward a stable solution. Each perturbation is accepted if the objective value is improved (reduced) or slightly degraded (increased). An increase in the objective value may be accepted early in the inversion, because this allows the algorithm to escape a local minimum in the objective value (Deutsch and Journel, 1992). For the MPI, we allow a large proportion of pixels to change populations for early iterations and reduce this proportion for later iterations.
The process of updating the spatial distribution of populations and adding perturbations to each population is repeated iteratively until the traveltime residuals increase for several consecutive iterations. When this occurs, the proportion of pixels allowed to change populations is reduced and the inversion is restarted using the slowness model with the minimum traveltime residual. If the objective value is not improved by this change, the estimate with the minimum traveltime residual is taken to be the optimal slowness field for the specified number of populations.

If multiple cross-sections are available in a region, the MPI can co-invert all the cross-sections to obtain a consistent estimate of the multi-2-D structure of the region. To determine if populations are likely to be the same for each cross-section, the MPI can be applied to each cross-section individually. If the region appears to have a small number of slowness populations, co-inversion of the travel times with the MPI may improve the regional consistency of the estimates.

**Interpretation of MPI estimates.**—The final multipopulation estimates can be used as starting models for an unconstrained-value tomographic inversion. This confirms the MPI estimates and allows smaller perturbations to be added to the estimated slowness populations. Iterative tomographic inversions work best when the perturbation from the starting model is small because the predicted raypaths are closer to the true paths. Thus, in cases where distinct lithologies exist, the updated MPI estimates may better describe these heterogeneities than an estimate from unconstrained-value tomography with a homogeneous starting model.

In cases where the ultimate goal is to estimate fluid flow properties or determine the lithologic structure of a subsurface reservoir or aquifer, the MPI offers significant advantages over many commonly used inversions. Tomographic inversions generally place few constraints on the slowness values, resulting in relatively smooth slowness models with inversion artifacts. The MPI improves structural interpretations and reduces inversion artifacts, especially in environments with a small number of slowness populations. The tradeoff for improved spatial resolution is reduced resolution in velocity variability.

In the following sections of this paper, we demonstrate the MPI with both synthetic and field crosswell P-wave travel times. The synthetic travel times were simulated through a slowness field with two dominant populations and added correlated variability. The field data were collected by Lawrence Berkeley Laboratories using multiple well pairs near the Kesterson Reservoir in California’s Central Valley (Vasco and Majer, 1993).

![Figure 2](image-url)

**Fig. 2.** Comparison of (a) the synthetic slowness field to the models estimated using: (b) unconstrained-value tomography, (c) the split inversion method (SIM), and (d) the multiple population inversion (MPI).
RESULTS FOR A SYNTHETIC AQUIFER

We compare the multiple population inversion (MPI) to both the split inversion method (SIM) and unconstrained-value tomoography using a previously published synthetic data set (Hyndman et al., 1994). This data set, composed of crosswell seismic traveltimes, was created by simulating the propagation of $P$-waves through a synthetic seismic slowness field (Figure 2a). This synthetic field is based on a fluviodeltaic aquifer in California’s Central Valley (Miller et al., 1990). The large zones represent interbedded clay, silt, or gravel lenses in a sandy aquifer. A field of spatially correlated deviates was added to the zonal slowness field to represent smaller scale heterogeneities. These deviates were calculated by multiplying the Cholesky decomposition of an exponential covariance matrix by a vector of standard normal deviates (Alabert, 1987; Johnson, 1987). Traveltimes were then calculated using equation (3a) along curved raypaths that were simulated from each source to each receiver through the synthetic slowness field.

To compare the different inversion methods, we used the same starting slowness field and ray-tracing parameters for all cases. Figure 2 illustrates the estimated tomograms from the three inversion methods relative to the synthetic slowness field. The tomograms from our zonal algorithms, SIM and MPI, provide better estimates of the geometry of dominant lithologies than unconstrained-value tomography. The unconstrained-value tomogram in Figure 2b also illustrates several inversion artifacts, such as the incorrect lateral variations in slowness in the mid-depth region of the tomogram. These artifacts make lithologic interpretation more difficult than possible with the zonal estimates.

Table 1 quantifies the results using the SIM and the MPI in this synthetic aquifer. The SIM results in a slightly smaller percentage of pixels assigned to the wrong population, while the MPI estimate has a lower traveltime residual. Both zonal methods generate accurate estimates of the geometry of the high slowness zones for this synthetic aquifer. These methods thus provide information about the dominant scale of heterogeneity in this aquifer. The main region where the zonations differ is in the upper right where both the MPI and unconstrained-value tomography incorrectly predict that the high slowness region intersects the well on the right. This is caused by poor ray coverage in this region where all the synthetic raypaths intersected the high slowness lithology. Smoothing constraints in the SIM reduces the slowness in this region because of low slowness estimates above and below.

Both zonal methods also provide accurate estimates of the average zonal slowness values. The estimated low slowness value obtained from the MPI is more accurate than that obtained from SIM (Table 1). The MPI high slowness value is a good estimate of the true harmonic average, and the SIM value is a good estimate of the true arithmetic average.

One advantage of the MPI, relative to the SIM, is improved computational efficiency. For each SIM iteration, rays are traced first to estimate the unconstrained-value tomogram, then rays are traced through many potential zonations of this tomogram to find the zonation (i.e., the slowness split values)

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<th>Table 1. Comparison of results for the split inversion method (SIM) and the multiple population inversion (MPI) using a synthetic data set.</th>
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<tr>
<td>Average traveltime residual per ray (μs)</td>
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<td>% mis-assigned pixels</td>
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<td>High slowness value (μs/m)</td>
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<td>Low slowness value (μs/m)</td>
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that minimizes traveltime residuals. An average of 13 additional ray-tracing steps were used to assess the optimal zonation for each SIM iteration. In contrast, each MPI iteration involves tracing rays only once.

Figure 3 illustrates the convergence of the MPI (with 25% of the pixels allowed to change populations), the SIM, and unconstrained-value tomography for this synthetic aquifer. The minimum traveltime residuals were obtained using 16 iterations of the MPI, with one ray tracing and back projection step for each iteration, and six iterations of the SIM. Since each SIM iteration involves an average of 14 ray-tracing steps compared to only one for MPI, the MPI is approximately 5.25 times more efficient computationally than the SIM for this example. After the 16th iteration, the MPI began to diverge slightly (Figure 3), and lowering the percentage of pixels allowed to change populations did not improve the objective value. The estimate from the 16th iteration was thus taken as the optimal tomogram (Figure 2d) for this method.

The convergence of the MPI is faster than unconstrained-value tomography for all but the first iteration, and the MPI converges to a solution with a lower traveltime residual (Figure 3). The convergence of the MPI can be sped up by increasing the proportion of pixels allowed to change populations or multiplying the median perturbation by a value greater than one, but this may result in a degraded objective value (i.e., larger average traveltime residual). Although the MPI is not very sensitive to this proportion, inversions were completed using several proportions to determine the best value within 5%. Allowing 25% of the pixels to change populations, provided the minimum traveltime residuals for this synthetic aquifer. Figure 3 also illustrates that the SIM converges to approximately the same average traveltime residual as obtained using unconstrained-value tomography. The dramatic divergence of the SIM in the seventh iteration was caused by overestimating the slowness of the embedded zones and as a result, greatly misestimating the zonation.

**RESULTS FOR THE KESTERSON FIELD SITE**

We also used MPI to co-invert traveltimes from seven crosswell seismic surveys collected through a near surface aquifer in California's Central Valley. This unconsolidated shallow aquifer, near Kesterson Reservoir, was deposited by the San Joaquin River as sequences of sand, silt, and clay (Benson, 1988). Vasco et al. (1993) and Vasco and Majer (1993) performed tomographic inversions of a portion of the same traveltime data. They found that a high seismic slowness region in their tomogram roughly corresponded with a region of high groundwater flow velocity, identified using a tracer test at the site. We also found a positive correlation between our seismic slowness estimates and permeability estimates for this site.

A dense well network was set up to study the distribution of selenium in the region's groundwater (Benson, 1988). The site used for this study (Figure 4), which has wells spaced from 1.5 to 14 m apart, is a subsection of a much larger well network. To predict the transport of selenium in groundwater, the subsurface lithologies need to be identified, and groundwater flow and solute transport properties need to be estimated for these lithologies. The MPI is applied to this field site to estimate the seismic slowness structure of the aquifer along the available planes of seismic data.

To determine the number of dominant slowness populations at the Kesterson site, we inverted each cross-section first independently with the MPI. We used the same homogeneous starting model, with the regional average seismic slowness of 577.7 $\mu$s/m for each inversion. This value was determined using a Householder transform to invert the system of traveltime equations describing straight raypaths through all seven crosssections. The single cross-section inversions, which are not illustrated here, indicated that the Kesterson traveltimes could be co-inverted for three regional slowness populations.

The tomograms from the MPI co-inversion are illustrated in Figure 5b, along with the unconstrained-value tomograms in Figure 5a. These tomograms illustrate the nature of heterogeneity in the Kesterson aquifer by unfolding the tomograms into a plane, as described in Figure 4. These slowness images indicate that the aquifer lithologies are much more continuous in the horizontal dimension relative to the vertical dimension, but the aquifer is not perfectly stratified. The general pattern of heterogeneity is the same from both MPI and unconstrained-value tomography. The tomograms from both methods show reasonable correlation at some wells, such as well 5; however, other wells such as 8 show poor correlation at tomogram intersections. This lack of correlation is likely to be caused by either traveltime picking errors or incorrect measurements of well offset or deviation. If well logs had been available, the slowness values could be fixed at the wells, thus eliminating this problem. Since the MPI slowness values are constrained, we can estimate the slowness tomograms without smoothing, which is often used to reduce inversion artifacts. The MPI can thus locate and describe smaller scale variability than tomographic methods that rely on smoothing for stability.

The initial convergence of the MPI is slower than unconstrained-value tomography for the Kesterson inversion, but both methods converge to approximately the same average absolute traveltime residual on the tenth iteration (Figure 6). By constraining the slowness to just three populations with the MPI we have parsimony. With no other available information, the constrained slowness model from the MPI provides a reasonable alternative to the unconstrained-value slowness model for regions where distinct lithologies are expected.

The MPI's convergence rate increased after the third iteration (Figure 6) because a third population was introduced. For the first three iterations, 30% of the values were allowed to change to two populations, then 10% of the values were allowed to change to three populations for the remainder of the iterations. These percentages were determined using sensitivity analysis, and we found that the inversion was rather insensitive to these parameters. The significant decrease in residuals resulting from the addition of a third population indicates that this is likely a reasonable addition to the slowness model. The convergence of the highly constrained MPI to approximately the same average traveltime residual as unconstrained-value tomography provides an additional indication that three populations describe the site's predominant heterogeneities.

The histogram of slowness perturbations is narrowed dramatically using the MPI. Figure 7a illustrates this histogram of slowness perturbations for all pixels of the homogeneous starting model, and Figure 7b illustrates the histogram for all pixels of the three population model. This final relatively symmetric distribution of perturbations contains only small values clustered around zero, again indicating that the three estimated
seismic slowness populations adequately describe this region of the Kesterson aquifer.

The three population slowness model was then used as a starting model for unconstrained-valued tomography. The resulting estimates (Figure 5c) preserve the general large-scale structure and add some smaller perturbations to the estimates. The most notable differences between Figures 5a and 5c are in the cross-section from I1 to I2, which is expected since this cross-section has the least raypath coverage, thus the greatest uncertainty. The updated three population estimates appear geologically reasonable for alluvial deposition.

**DISCUSSION AND CONCLUSIONS**

In this paper, we have developed the multiple population inversion (MPI) to estimate the spatial distribution of a small number of seismic slowness populations and the mean slowness values for each population. We have presented the application of the MPI to crosswell seismic $P$-wave arrival times for both
synthetic and field data. We have shown that the MPI is a useful method to image the structure of dominant seismic slowness populations using traveltime data.

For the synthetic aquifer, the MPI accurately characterized the structure of two slowness populations, resulting in a smaller average traveltime residual than obtained from unconstrained-value tomography. Reasonable constraints on the slowness model actually reduce the objective value (i.e., the average traveltime residual) for this case. We compared the MPI with the SIM, which has a flexible objective function that can combine multiple data types. The SIM also provided an accurate estimate of the lithologic geometry for this synthetic data set; however, the SIM is less efficient, more difficult to implement as a regional co-inversion of multiple cross-sections of data, and relies on more assumptions than the MPI.

For the shallow Kesterson aquifer, the MPI was used to co-invert traveltimes between seven well pairs to provide a regional three-population slowness estimate. For comparison, unconstrained-value tomograms were estimated independently for each cross-section. Both methods converged in ten iterations to approximately the same traveltime residual. The MPI imaged finer scale structures, which could be thin zones that influence fluid flow, than unconstrained-value tomography that relies on smoothing the slowness model.

The application of crosswell seismic techniques to shallow aquifers appears promising, although extreme care must be taken to survey the well field accurately. Unlike petroleum applications that involve distant wells, this site has well offsets of 1.5 to 14 m and minor errors in the measurement of offset between wells can have significant effects on the estimated slowness values. At the Kesterson site, the distance between wells was measured with a tape, and the wells were assumed to be deviated but straight. Because of the inaccuracy in these measurements, the offset between several well pairs had to be adjusted slightly. We determined that very minor adjustments to the well offsets improved the coherency between cross-sections and reduced the traveltime residuals in the co-inversion. The largest adjustment was just over 1% of the well offset, which is within the range of measurement error.

Rather than adjusting the offset between wells, Vasco et al. (1993) and Vasco and Majer (1993) added small anisotropy corrections to the traveltimes for their inversions at this site. A homogeneous anisotropy correction has the same effect as adjusting the offset between wells. If necessary, the MPI can account for velocity variability associated with anisotropy by estimating an anisotropy coefficient for each identified zone. This would provide regional estimates of both slowness and anisotropy coefficient and could account for variability in the intrinsic properties of different lithologies that are manifested in the zonation as anisotropy. For example, variations in clay content could be captured by the anisotropy coefficient.

The next stage of this research will be to combine tracer data with seismic traveltimes to estimate the 3-D structure of the Kesterson aquifer. If no tracer data were available at this site, estimates of the hydraulic properties would be derived from either core data or local pump test data. Well log data would also be useful to verify the estimates at the well, although this information is not available in this region of the

\[ \text{Fig. 6. Comparison of mean traveltime residuals for the Kesterson Field data using the multiple population inversion (MPI), unconstrained-value tomography (UVT), and UVT to update the MPI estimates.} \]

\[ \text{Fig. 7. Histograms of spatially distributed slowness perturbations (a) before and (b) after the multiple population inversion.} \]
Kesterson aquifer. Future papers will examine other aspects of this estimation problem, such as: (1) generating 3-D seismic slowness realizations from multi-2-D estimates using geostatistics, (2) combining seismic and tracer information to estimate the lithologies and the 3-D permeability field at the site, and (3) inferring the relation between seismic and hydraulic properties using field scale measurements.

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